The 2022 MidMCM®

Kayla Blyman, MidMCM Director

Congratulations to all the teams who participated in the second annual international Middle School/Level Mathematical Contest in Modeling (MidMCM)®. A total of 70 middle school/level teams, with up to four students each, representing 46 schools and 8 countries/regions, competed in MidMCM this year. The judges were impressed with the obvious time and effort that all teams put into their submissions. The MidMCM is an amazing and rewarding experience for students, advisors, schools, and judges around the world. Laura Celik, an advisor for three teams from Princeton Charter School in New Jersey, USA, provides a reflection on the experience she and her students had with this contest in *Reflections from a MidMCM Advisor* in this issue of Consortium.

We applaud all participants. In the end, three teams rose to the top and were designated Outstanding Winners. We are excited to have once again have the National Council of Teachers of Mathematics (NCTM) support the designation of one of our MidMCM teams as the NCTM Award winner.

Outstanding Teams

- 12704 Wenzhou Nanpu Experimental Middle School, Zhejiang, China
- 13296 Princeton Charter School, NJ, USA (NCTM Winner)
- 13338 NUS High School of Math and Science, Singapore

Kayla Blyman, Ph.D., is assistant professor of mathematics at Saint Martin's University In Lacey, WA. She is also the ICM director.

The 2022 Contest

We are truly impressed by the amazing teamwork and creativity shown by participating students. The MidMCM problem, *Polygon Paradise Park*, challenged teams to balance the different interests, priorities, and restrictions given for each member of a group of four friends to plan a day at an amusement park that was most enjoyable for the group. The variety of approaches that teams took to the problem delighted the judges.

Overview

While COMAP has offered international modeling contests for over 40 years, in 2022 MidMCM joined HiMCM as an annual fall contest for the second time. As increasing numbers of schools engage their students in mathematical modeling, we see continued increasing participation in COMAP's modeling contests. The HiMCM started with 115 students and over the course of 25 contests has had 44,544 students apply their mathematical knowledge and skills as they modeled challenging contest problems. We anticipate the MidMCM will similarly grow in numbers of students and teams as we saw evidence of that growth from our first year with 54 teams to this year with 70 teams.

The mathematical modeling ability of our MidMCM participants is evident in the problem solutions and professional submissions we receive. We acknowledge and credit advisors and teachers for their work with these students. As teachers and students engage in mathematical modeling at all levels, we are happy and excited to assist your efforts. Let us know how COMAP might support your modeling activities.

Results

Table 1 shows the results of the 2022 Middle School/Level Mathematical Contest in Modeling (MidMCM).

In total, 258 students participated in the 2022 MidMCM. A wide range of schools competed, including teams from Australia, Canada, China, Philippines, Singapore, South Korea, the United States of America, and Vietnam. The 14 teams from the United States represented 9 states. Submissions included 56 foreign teams with China representing about 79% of those teams.

Of the 258 student participants this year, 83 (32%) self-identified as female, 128 (50%) self-identified as male, and 47 (18%) did not specify gender. Our MidMCM participants are quite diverse. We hope that all students enjoyed their contest experience and continue to pursue further Science, Technology, Engineering, Arts, and Mathematics (STEAM) in their studies and beyond. We welcome all levels of middle school/level students to participate in the MidMCM. The only prerequisites for success in the contest are middle school/level mathematics skills and concepts and a desire to work on a team to solve a real-world problem.

Rules

Teams can schedule their own working time within the contest window. Our 25-page limit for papers results in well-written and well-organized solution papers. Teams must choose what information, modeling, and graphical support is necessary to present their full analysis of the problem within 25 pages.

Outstanding	%	Finalist	%	Meritorious	%	Honorable Mention	%	Successful Participant	%	Total
3	4%	4	6%	8	11%	20	29%	35	50%	70

***Table 1:** 2022 MidMCM Judging Results*

One important rule is that students may only use the members of their own team along with inanimate (non-living) sources to complete the contest problem. Students may not use their teachers or advisors, any chat rooms, electronic communication, or social media sources. Each year we have some teams that do not understand this rule. To be clear, contacting an expert in a field or an author of one of the referenced sources is a violation of this rule. Gathering data from persons outside of your team through the use of an interview or a survey or a questionnaire is a violation of this rule. Using solutions shared electronically by other teams or by organizations is a violation of this rule. Again, only the team members may contribute to the solution through their own knowledge and work, and by using inanimate resources (e.g. research articles, web sites, textbooks, journals, publications). Additionally, COMAP will never require that you purchase additional materials or information to be successful in the MidMCM. The materials and information provided by COMAP, along with your own team's knowledge, skills, and perhaps a bit of research using allowed references, is all that you need for success.

COMAP uses Twitter and Weibo to provide contest information to participants. Follow us @COMAPMath on Twitter or COMAPCHINAOFFICIAL on Weibo for contest guidance and up to date contest information.

Judging

All contest submissions are electronic. This allows a high quality and diverse judging pool to simultaneously judge papers. In December 2022, we conducted our first round of contest judging with judges from the U.S. states of New Jersey, New York, Ohio, and Washington. Each paper is read and scored by two preliminary judges. We thank these judges for their careful review of our MidMCM submissions.

All judging is blind with respect to any identifying information about the participants or their schools. Preliminary judges rank papers as Finalist, Meritorious, Honorable Mention, and Successful Participant. Judges sent all papers ranked as "Finalist" or "Meritorious" to Final Judging. This year, 15 papers went to Final Judging for a panel of three judges to consider. These 15 papers were the best submissions from the preliminary round. At final judging the judges decided which papers were designated as "finalist" and from those chose the *"best of the best"* as Outstanding papers. Three papers earned the Outstanding award with four more being designated as Finalists. We feel that the structure of preliminary and final judging provides a good process for identifying our top papers.

MidMCM®

The Future and The New MidMCM

For 25 years, the HiMCM has sought to provide all high school students the opportunity to compete and achieve success in applying mathematics. COMAP's mission of improving mathematics education for students of all ages was the motivation for offering the first MidMCM in 2021. Our efforts remain focused on meeting this important mission. Mathematical modeling continues to grow at all school levels around the world, and we recognize that middle school students are now modeling too. The MidMCM will continue to occur concurrently with HiMCM in the fall each year. The MidMCM allows middle school/level students the opportunity to demonstrate their mathematics and modeling abilities. Please visit **www.midmcm.com** for more details and the results of this contest each year.

The MidMCM and the HiMCM provide a vehicle for using mathematics to build models that allow students to represent, and to understand, real world behavior in a quantitative way. Both contests enable student teams to look for patterns and think logically about mathematics and its role as a language in our daily lives. Students gain confidence by tackling ill-defined problems and working as part of a team to generate a solution. We are excited that in our contests, applying mathematics is a team sport.

Advisors and students often ask what level of mathematics is required, and what special programming or coding skills are needed for the contest. Our MidMCM problems require only middle school/level mathematics. As in all of our contests, each of our problems is accessible on multiple levels. Students should apply the mathematics they understand and are able to explain in their solution analysis.

Advisors need only be motivators and facilitators to encourage students to be creative and imaginative. COMAP encourages all middle school STEAM faculty to get involved, encourage your students to be problem solvers, make STEAM relevant, and open the doors to future success. We want to partner with teachers as we continually strive to improve the contest and make it accessible and impactful to all students. Any school can enter, and each school can enter as many teams as that school desires. MidMCM has no restrictions on the number of total schools or the numbers of total teams from any one school or overall.

More than just learning skills and operations, mathematics is both an art and a science. Through mathematical modeling, students learn to think critically, communicate effectively, and be confident, competent problem solvers. Success is not only about the procedural technique used, but the conceptual understanding in discovering the role of assumptions and model development in driving those techniques to a valid solution and conclusion. The ability to recognize problems, formulate a mathematical model, use technology, and to communicate and reflect on one's work are important skills to develop. Applying the mathematical principles and concepts that one learns is key to individual and societal future success.

~ Awards ~

Outstanding
Finalist
Meritorious
Honorable Mention
Successful Participant

After final judging, MidMCM papers receive a designation in one of the categories above. Depending upon the quality of the papers, the top 20-25% of submittedpapersreceive a designation of Meritorious or above, with approximately the top 1-2% designated as Outstanding.

2023 Contest Dates

Mark your calendars for the third annual MidMCM, and the 26th HiMCM, to be held November 1 - 14, 2023. Registration for the 2023 MidMCM and HiMCM will open in September. Student teams may work at any time during the contest window. At the team members' convenience, teams download and choose their problem, complete their modeling solution, and electronically submit their solution document by the deadline on November 14th. Again in 2023, one team for each problem will receive the NCTM award. Teams can learn more about COMAP's contests and registration at :
www.midmcm.com.

MathModels.org

Powered by COMAP content, MathModels.org is a wonderful resource for students and teachers to make mathematical modeling a year-round activity. Teachers and students may use the materials found on this site to enrich their classes and help prepare students for mathematical and interdisciplinary modeling contests. We encourage you to visit **www.mathmodels.org**.

Problem Discussion

We were once again extremely pleased with the submissions in the second annual MidMCM. The teams followed the guidance and submitted well-written and organized papers with appropriate mathematics, explanations, and diagrams.

Our MidMCM problem was accessible to students using only middle school/level mathematics. Some teams attempted to use advanced concepts and tools found on the Internet that they did not explain clearly or use appropriately. Judges recognized this, and these papers did not do well. We are not looking for papers that use the most advanced mathematics. We have found that the best papers develop a mathematical model that incorporates middle school/level mathematical concepts and tools that the teams understand, are able to fully explain, use appropriately, and analyze subsequent results. The most important aspects of solutions are the model development and the clear use and analysis of the model toward addressing the requirements of the problem.

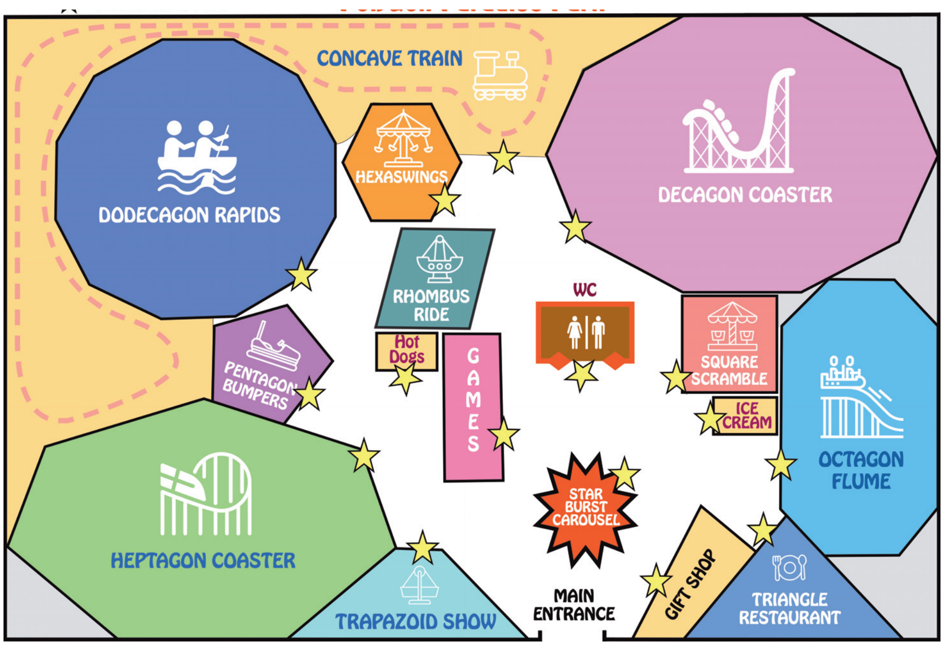
The specific problem discussion below provides comments on how teams addressed the requirements of the Polygon Paradise Park problem. Following this section, we provide judges' comments about the solutions and presentations by breaking down the various parts of a submission. To view the complete problem statement, visit
www.mathmodels.org or
www.contest.comap.org/highschool/contests/himcm/previous%20problems.html.

Problem C: Polygon Paradise Park

Anytime a group of people spend time together there is a certain balancing of preferences that must occur for everyone to have a positive experience. This is most apparent when traveling or attending events where choices must be made. We asked MidMCM teams to do the balancing work for a group of friends visiting Polygon Paradise Park. As is the case with most groups of people, the group of friends described in the problem had diverse interests and requirements for their day at Polygon Paradise Park.

Overall, the MidMCM problem asked teams to create a schedule for the day that would be most enjoyable for the group of four friends described. After developing the schedule for the group,teams were asked to share their schedule with the group of friends in a letter explaining why it is the best schedule for the friends. Finally, teams were asked to reflect on their modeling experience by thinking about how they could apply their process to create a schedule for their MidMCM team to spend a fun day at Polygon Paradise Park.

While the problem statement provided general descriptions of requirements and preferences of each of the four friends, many teams chose to expand upon these descriptions. Teams then needed to determine what each of the friends would enjoy doing at Polygon Paradise Park based upon the resulting description of each. Additionally, a scaled map of the park was provided along with waiting times throughout the day and event durations for each of the places shown on the map. Neither the problem statement nor the additional information provided gave travel times from one location to another. Teams needed to make an assumption about the speed the group of friends would walk and to use the map scale and measurements they made to determine how long it would take to get from one location to the next.



Map of Polygon Paradise Park

After figuring out what an ideal day would look like for each of the friends individually, teams needed to figure out how to merge the days of all four friends so that each would feel like they had a great day at Polygon Paradise Park. The problem statement allowed for the group of friends to spend four hours of the day as two groups of two instead of as a group of four and most teams chose to do just that. As teams constructed their schedule for the day, they needed to not only take into consideration the preferences of the friends, but they also needed to consider wait times throughout the day as well as the time it would take the friends to walk from one location to the next.

Once teams had developed a schedule, they were required to write a one- to two-page letter to the group of friends that described the highlights of their recommended schedule and explained why their schedule offered the best possible day at Polygon Paradise Park for the group of friends. This was an opportunity for teams to communicate their modeling to a group of their peers. Additionally, it was a chance for them to give insights to considerations they had chosen not to include in their modeling by addressing them in the argument for why their schedule produced the best day. As a bonus, many teams showcased their creativity quite well as a part of this letter.

Finally, teams reflected on their modeling by thinking about how the best schedule for their MidMCM team would differ from the one they developed for the group of friends presented. Teams were <u>not</u> required to create a schedule for their MidMCM team, but they were asked to reflect on how the process they used to develop the schedule for the group of friends given could be applied meaningfully to develop a schedule for their MidMCM team. Some teams chose to develop a schedule for their team, while many others stopped at comparing themselves to each of the four friends described in the problem statement. The most meaningful reflections more thoroughly explored the generalizability of both their model and their modeling process.

Judges saw many interesting papers in the second MidMCM. Teams presented a range of methodologies, from simple and straightforward to more complex, to respond to the requirements of this problem. It seemed teams had fun developing their schedules and the judges were happy to see their creativity shining through.

Judges' Discussion

While the problem discussion above provides comments on the solutions to this year's problem, in the following paragraphs we examine the sections of a submission and provide comments about the solutions and the presentation of the solutions from our judges' points of view. At the end of the article, we have included excerpts from our Outstanding papers as exemplars. Mathmodels.org members can view all the unabridged versions of the Outstanding papers online.

Overall

Participants must ensure their papers follow the contest rules posted on the contest website. Papers that are coherent, organized, clear, and well written provide a great impression to the judges. The logic and mathematics of these papers are easy to follow. Teams should present their entire submission in 25 pages or less, using at least 12-point font. These 25 pages should include your introduction/executive summary, your solution that addresses all requirements, a resource list, and any appendices. While students may want to include some background research on the problem topic, this information should be brief. It is not the number of pages, but the ability to complete all problem requirements and communicate the solution in a concise and articulate fashion that will merit recognition. Students should use spelling and grammar checkers before submitting a paper. Foreign papers should ensure that all symbols in tables and graphs are in English. Student and school names should not appear on solution papers.

Papers considered for Finalist and Outstanding start with a clear summary that describes the problem. These papers then preview their paper with an organized Table of Contents. They present assumptions with justifications, explain the development of their model and its solutions, apply their model, and support the results mathematically. These best papers communicate all the above clearly, do a sensitivity analysis, address strengths and limitations, and finally, close by stating overall conclusions.

As we did for the first MidMCM, we continued to offer students and teachers the following guidance to assist in organizing a problem solution submission for the second MidMCM.

Guidance for MidMCM

Solutions must be in PDF format and submitted in one PDF document. However, this does not preclude MidMCM teams from doing mathematics, graphs, tables, sketches, etc. by hand and including pictures of their work in the single PDF document submission. As students move to high school and the HiMCM, we expect that submissions will be typed. For the MidMCM, advisors may technically assist students in putting their solution components into one PDF format file for submission.

As with HiMCM, there is a 25-page limit for the submission document. This does not mean your solution must be 25 pages. A shorter submission is certainly acceptable. All portions of your submission (text, graphs, tables, charts, pictures, etc.) must be within **one** PDF document that is 25 pages or less. We accept partial solutions.

In general, a complete solution submission is organized as follows:

Executive Summary – Write this summary after you have done all your work. This one-page summary is Page #1 of your solution document. It provides an overview of your work and includes actual results.

Table of Contents – List the major items in your solution document to show the organization of your paper.

Introduction and Restatement of the Problem – Introduce the problem. Restate the problem and requirements in your own words.

Assumptions with Justifications – State any assumptions you made to simplify and solve the problem and state why you made those assumptions.

Variable Definitions – Define any variables you use in your model and equations.

Presentation of Model and Solution – Ensure you address all requirements and describe what you are doing in solving the problem. Show and explain all your work. Use representations that help you tell the reader how you solved the problem (for example: equations, tables, graphs, pictures, etc.).

Analysis of Your Work – Address any strengths (good points) and limitations (weaknesses) of your model and solution.

Concluding Paragraph – End your solution paper with a final concluding paragraph that summarizes your results and/or makes recommendations for future work.

Reference List – List any sources that you used to solve the problem (for example, website pages, newspaper, or magazine articles, etc.)

Executive Summary

As the first page, or cover sheet, the Executive Summary provides a first impression of your paper. It offers the judge (or any reader) not only a synopsis of the paper and your modeling and analysis process, but also the solution to the problem. Judges see many well-detailed descriptions of the problem and the process but look for well-written and complete summaries that include the important actual results and recommendations. Teams should write the Executive Summary after they finish their solution to summarize the entire contents of the paper. Although written last, ensure you spend time on this important part of your submission. **Example 1** presents a good summary.

Introduction/Background

Following the summary, teams should provide some brief background, restate the problem, and perhaps provide a preview of the solution. Teams should not simply repeat what was in the summary. Most teams this year limited their background to a restatement of the problem. In **Example 2**, a team restates the problem, choosing to expand on some parts while collapsing others.

Assumptions with Justifications

Good models make necessary assumptions to help simplify the modeling process. These are sometimes called simplifying assumptions. A common mistake judges see is teams that make assumptions that are not needed nor relevant to developing their model. Good and relevant assumptions are difficult to identify and articulately state. You should include a short justification to show each assumption is reasonable and necessary. **Example 3** gives one of the longest assumptions sections we saw this year. However, it was not so long that it detracted from the modeling. This is due to the team's use of all these assumptions in their model, making them all truly necessary, and the thorough justification of each which contributes greatly to the length of the section. The judges encourage teams to be discerning in their choices of assumptions.

Definition and Use of Variables

Most mathematical models include several (and sometimes many) variables that teams must define for the reader. This list of variables should include the variable symbol, a short description of the variable, and the units of the variable. Using best practices, teams should focus on a manageable set of variables when modeling. Additionally, as you use variables in your model, remind the reader of the variable definitions and units. This practice assists the reader in following the logic of your process. A nice list of variables for is shown in **Example 4**.

Mathematical Model

The development of the mathematical model is the most important part of your submission. There is always more than one appropriate solution method to our MidMCM problems and so teams should address the problem with the mathematics they know and understand. Papers should explain the development of the mathematical model(s) and/or algorithm(s) and define all variables in a logical manner. Teams should take the reader on a journey describing their thought process and steps. Better teams will explain why they choose their model and how they plan to use or modify it to fit this problem. Judges are more impressed with a well thought out (and perhaps simple) model then with a very complex model that a team struggles to apply. To impress the judges, focus on applying sound principles to your model that you understand. Judges do value creativity and thinking "outside of the box" in your modeling process but be sure to balance creativity with your level of expertise and modeling experience. So, be creative and have fun. **Example 5** illustrates the power of using simple mathematics to address the Polygon Paradise Park problem.

There are many ways to model and analyze the MidMCM problem. This year we saw a variety of appropriate, as well as creative, models. Many times, a picture is worth a thousand words! The use of tables, graphs, and images is often helpful to show your modeling process.

The Letter

The purpose of the letter is to show the judges that regardless of the complexity of your mathematics or analysis, you can convey your work and solution in common terms. Often, the recipients and consumers of mathematical modeling and research are not as technically astute as the mathematicians and scientists doing the work. In writing the letter, teams must translate their hard work into a shorter descriptive format. This increases the likelihood your audience will understand and consider your solution and recommendations.

Like your summary, the letter briefly describes the problem and background, your model or process, and your results and any recommendations. Some teams include visuals in their letter. These graphics can assist the team in presenting their process and the reader in understanding the model and results. **Example 6** provides an example of a creative strong letter.

Additional Details

Sometimes teams go beyond the requirements of the problem in a way that shows how deeply they are thinking about the problem. These details are always appreciated by judges when they come in addition to, and not at the expense of, the problem requirements. **Example 7** provides one example of how a team did that on the Polygon Paradise Park problem by analyzing their resulting schedule for each of the four group members. In doing so, the team ensured that their final schedule was going to be a positive experience for each of the four group members and they shared with their readers how they knew that was going to be the case.

Strengths and Limitations

As teams only have a short time to develop their solutions, they should be critical of the approaches and mathematics they used in their solution. Teams should identify strengths and limitations of their work. Is your solution reasonable? Under what conditions will it perform best and where will it not? Additionally, teams should address how changes might impact their solution. Was there a critical assumption made that if changed would significantly change the solution? Would changing the values of any constants or multipliers (coefficients) significantly impact your results? Teams should validate their models, even if by numerical example or intuition. **Example 8** gives an example of a thoughtful discussion of strengths and weaknesses. Few teams included a sensitivity analysis – an exploration of the impact of an assumption by changing it in some way. A fantastic assumption to explore in this problem for many teams would have been walking speed. What happens if the group moves faster or slower than you assumed they would?

Reflection

The purpose of the reflection part of this problem that asked teams to examine if their process could be replicated to develop a schedule for their MidMCM team visiting Polygon Paradise Park was to have teams think about the adaptability of their model. Few teams discussed this beyond the superficial details of their team member's preferences. The few that did manage to start addressing the question of adaptability. **Example 9** is from a team who started to do just that after addressing some of the more superficial details.

Conclusion

A clear conclusion and answers to the specific scenario questions are key components to an Outstanding paper. Attention to detail and proofreading your paper prior to final submission are vital as the judges look for excellence in your submission. Teams should take the time to read the entire paper to ensure they have followed a logical sequence and flow that tells a story from start to end. Few teams included a conclusion like this, but several included a conclusion that was a meaningful reflection on their work, complete with ideas for the future. **Example 10** is one example of this type of conclusion that the judges saw often.

Citations and References

Citations are very important within the paper, as well as either a reference list or bibliography page at the end. Teams that use existing models should cite their source(s) within the paper at the point they present the model and include a reference citation in the back of the paper. This is also true for all graphs and tables taken from the literature. Use "in line" documentation with footnotes or endnotes to give proper credit to outside sources. All data, figures, graphs, and tables that come from outside sources require documentation at the point in the paper where they appear. Lack of documentation will result in a lower designation.

Judges have noticed many teams using Wikipedia as a source. While Wikipedia can be a great tool for learning about something rather quickly, it is not an academic source and as such should not be trusted as a sole source. Teams should look for and use more authoritative – ideally primary sources – whenever possible.

Final Thoughts

The submissions for the second year of the MidMCM impressed judges and left them excited for the future of the contest. We truly enjoy reading all solution papers. We hope all participating students also enjoyed the experience and improved their mathematics, modeling, and communication skills in the process. We encourage students of all levels to compete in future MidMCM competitions. To be successful, read the comments and guidance provided in this article, see the TIPS article on the COMAP website, and visit mathmodels.org to review previous problems. Follow us @COMAPMath on Twitter or COMAPCHINAOFFICIAL on Weibo for information about all COMAP contests.

List of Examples:

1. Summary (Problem C, Team 12704, WenZhou Nanpu Experimental Middle School, China)
2. Intro/Problem Restatement (Problem C, Team 13338, NUS High School of Math and Science, Singapore)
3. Assumptions (Problem C, Team 13296, Princeton Charter School, USA)
4. Variables (Problem C, Team 13338, NUS High School of Math and Science, Singapore)
5. Model Development (Problem C, Team 13296, Princeton Charter School, USA)
6. Letter and Plan (Problem C, Team 12704, WenZhou Nanpu Experimental Middle School, China)
7. Additional Detail (Problem C, Team 12704, WenZhou Nanpu Experimental Middle School, China)
8. Strengths and Limitations (Problem C, Team 13296, Princeton Charter School, USA)
9. Reflection (Problem C, Team 13296, Princeton Charter School, USA)
10. Conclusion (Problem C, Team 13338, NUS High School of Math and Science, Singapore)

Reflections from a MidMCM Advisor

Laura Celik

I first heard of COMAP and the MidMCM from a message on the American Modeling Teacher's Association (AMTA) email list. This was a few days before the registration deadline. I reached out to two of my students who had been active completing optional assignments and asked them if they would be interested in putting together teams to compete in the MidMCM. A few days later we entered three teams; two teams of 7th graders and one team of 8th graders.

I downloaded the problem statement as soon as it was available and shared it with my students. They were excited to find out they would be designing an ideal day at a theme park. It was the perfect topic for this age group. Most of the students had some personal experience to inform their work, and the choices they were optimizing between were varied and entertaining. The teams had two weeks to write a 25-page paper detailing their solutions to the problem. They met in separate rooms for 30 minutes every day during Study Hall to work on the contest. As their advisor, I supervised but did not give feedback. Each team had its own style and developed its own workflow.

One of the 7th grade teams was very vocal. Every time I entered their classroom, someone was up at the whiteboard explaining their ideas, and the rest of the team was shouting out their support or disagreement. They were so animated and intense that a few other teachers came to me, wondering what was going on. This team had a clear team leader who led the sessions, and another teammate who kept everyone on track when it came to meeting deadlines. One member said: "It was fun to work together. The process of finding a solution was fun." This team earned an Honorable Mention.

The other 7th grade team was quieter. It was composed of four students who do not typically work together. They took a while to warm up to each other and to figure out who would lead the group. Eventually they came together and completed the project. This team earned a rating of Successful Participant.

The team of 8th grade students had less Study Hall time to work on their project. These four students take two math classes (Geometry and Algebra II) and have more daily homework. Despite their limited time, they were the most successful team. For two weeks, they would briefly meet, discuss their progress and any decisions that had to be made by the group, divide up the work, work on it independently, and then share the results with each other. They had two large work sessions outside of school. This team had two clear co-leaders, with support from the other two teammates. They earned an Outstanding ranking and were also named the NCTM Winner. One of the team members said: "COMAP was a great experience that allowed us to understand what it meant to think about a problem on a larger scale and for an extended period of time. Not only did it develop our analytical skills but also our collaborative and organizational skills."

All twelve participants had their own unique experience with this project, but the overarching theme was growth. They were given a challenging problem and guidelines on how to present their solution, and then a set time each day to encourage their progress towards their goal. I would recommend MidMCM to any middle school teacher looking to provide a challenge to their students. The contest provides an opportunity for students to be team leaders, learn project management skills, and use skills they have learned in both math and language arts classes to create a unique solution that makes them proud.

Laura Celik is a 7th and 8th Grade Science Teacher at Princeton Charter School in Princeton, NJ. She can be reached at lcelik@princetoncharter.org

Summary

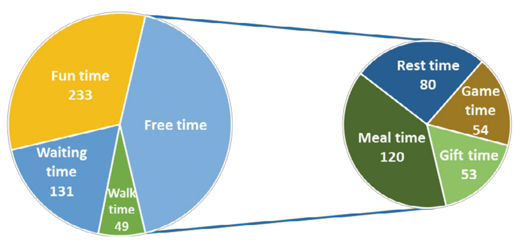
A group of four friends has hired our team at the MidMCM Travel Agency to make sure each member has a great time during their one-day trip to Polygon Paradise Park. Our team is to develop a schedule to make the group's day at the park the best possible.

There are 17 different locations in this park, we call them as X_i. First, we assign these 17 locations coordinate values in pixels, and use the Euclidean distance to calculate the pixel distance between any two locations. Then, we convert the pixel distance to walking time, and find the center location is X_{13}(Restroom, WC). Third, we transform the problem into a TSP problem, and find the shortest traversal path. Fourth, on the basis of the shortest traversal path, we divide the park map into 4 parts. Before the partitioning, 37% of walk times were 5 minutes or more. After partitioning, the walking time of the farthest two points is 5 minutes.

When develops the schedule, we arrange key locations first, and divide the 12 hours into four different sections, each section corresponds to a part of the park. After completing the development of the schedule, we gave each member of the team corresponding notes.

There are five highlights in our schedule.

- The average waiting time value for completing 10 rides only 7.5 minutes.
- The longest walking time not exceed 5 minutes.
- The schedule includes enough free time.
- Fully meet everyone's different needs.
- All rides and shows will be completed.



The time allocation arrangement in one day

Example 1: Summary Problem C,
Team 12704, WenZhou Nanpu Experimental Middle School, China

1.1 Problem interpretation

Our problem or 'challenge' is to plan a day of fun at Polygon Paradise Park for four friends. The park is open from 9am to 9pm. There are various activities in the amusement park, including rides of different types and thrill levels; a show with various performances throughout the day; food options, entertainments and other more basic facilities. We will have to meet the different requirements of the 4 friends to ensure maximum level of happiness. But no matter how perfectly we plan the activities, there will always be walking and waiting time. There is also a limitation that to maximise the enjoyment of the friends, it will be necessary to divide a 4 hrs period,

<u>Preference of the 4 friends</u>

-Ming cannot go on rides with extreme thrills or medium thrills after lunch. If not, he will throw up and severely lessen the enjoyment of himself and others. Ming also wants to win some prizes at the game's booth.

-Ishmael wants to try out as many roller coasters as possible, which includes extreme thrills, hence Ming and Ishmael cannot be together. He also wants to attend the magic show.

-Karine is very impatient and cannot wait in long lines which make splitting up her with Ming perfect as Ming cannot tolerate the extreme thrills and usually it's the extreme thrills that have long lines. She also wants to see the circus acrobatics show. And she has some allergy that makes her unable to eat hot dogs.

-Freya likes water rides and wants to spend time with friends. She also wants to have a souvenir to remember the trip.

1.2 Research motivation

-To develop a method to do scheduling while meeting the preferences of all friends

-To create a Mathematical Model to evaluate the scheduled cases enjoyment level

-To select the best trip schedule for the four group of friends to meet all preferences

Example 2: Intro/Problem Restatement Problem C,
Team 13338, NUS High School of Math and Science, Singapore

2 Assumptions

1. **Walking Speed: The walking speed for the group is constant and is 1.3 m/s.**

   Justification: This is the average walking speed given by the NCBI. (2) and this assumption of constant pace will help our task of creating a more straightforward and accurate model.

2. **Bathroom Usage: The group will have to go to the bathroom 3-4 times during the day trip.**

   Justification: According to our sources (3) , the amount of bathroom trips in 24 hours is around 4-10, the average of which is 7. Since the group is only in the park for 12 hours, we can say the average trip is around 4 times. (rounded from 3.5)

3. **Breakfast: The group ate breakfast before arriving at Polygon Paradise Park.**

   Justification: According to Northwestern Medicine, people should aim to eat breakfast within an hour of waking up. A survey conducted by Edison Research shows that most people wake up before 7:30, meaning the group of friends will most likely eat breakfast before 8:30. Our job is to provide a schedule for the group starting from 9:00 AM, so breakfast will not be considered as a factor for our model.

4. **Arrival time: The group arrives at the park before or at 9:00 AM**

   Justification: We are tasked with providing them a schedule starting at 9:00 AM and ending at 9:00 PM. This means that they are expected to arrive at 9:00 AM, and we are **only** responsible for providing the schedule between that 12 hour window.

5. **The ideal lunch time is from 11 AM to 2 PM (4-5 hours after breakfast).**

Example 3. Assumptions (Problem C, Team 13296, Princeton Charter School, USA)

Justification: The ideal lunch time according to multiple sources is 4-5 hours after breakfast, and 11 AM to 2 PM are the corresponding times assuming a breakfast from 7-9AM. (5)

6. **All measurements of time in minutes will be rounded UP to the nearest minute.**

   Justification: Traveling in an amusement park will never be perfect, and it is completely okay to omit half a minute for traffic and/or getting on rides.

7. **The group would prefer to eat together.**

   Justification: Eating is an inherently social activity and most people prefer to eat with others, as evidenced by a report from the University of Oxford. (4)

8. **Between given hours, wait times change at a constant rate.**

   Justification: It is unlikely that wait times simply jump in time from one hour to the next, so it is most practical logical to assume that wait times decrease at a roughly constant rate.

9. **Each member's preferences matter equally and are weighted in a schedule as such.**

   Justification: We are tasked with creating the most efficient, but also most enjoyable trip for the team. If we disregard the member's preferences, they would have a less enjoyable time at the park, and we are tasked to create a schedule that guarantees that everyone has a good time.

10. **The group would prefer to ride rides at times when the wait times are in the bottom quartile.**

    Justification: As evidenced by Karine's preference, humans are fundamentally impatient. Furthermore, minimizing waiting time allows the group's day to be

Example 3. Assumptions (Problem C, Team 13296, Continued Princeton Charter School, USA) maximally efficient. They can do more fun activities and rides instead of waiting in long lines, which is explained in an article cited (6).

11. **Each member of the group will want to try other activities than just their preferred activity.**

    Justification: The group members are not confined to their stated preferences and a complete amusement park is designed to contain variety.

12. **The arcade is open from 9 A.M. to 9 P.M. and has no waiting time.**

    Justification: The information provided doesn't say anything about when the arcade will be open, so we will assume the arcade is open from 9 A.M. - 9 P.M. In an arcade, there are waiting times for specific games, but no information is given about games or waiting times. We won't consider this as part of our model to make it less complex.

13. **It will take roughly 10 minutes for a bathroom trip**

    Justification : One of the articles cited (7), states that it takes around 20 seconds to actually go to the bathroom. However, considering washing hands, flushing toilets, and waiting in lines, we will say it takes around 10 minutes for a bathroom break. Giving lots of time ensures that if the line is longer, it won't affect the whole schedule.

Example 3. Assumptions (Problem C, Team 13296, Continued
Princeton Charter School, USA)

2.2 Variables

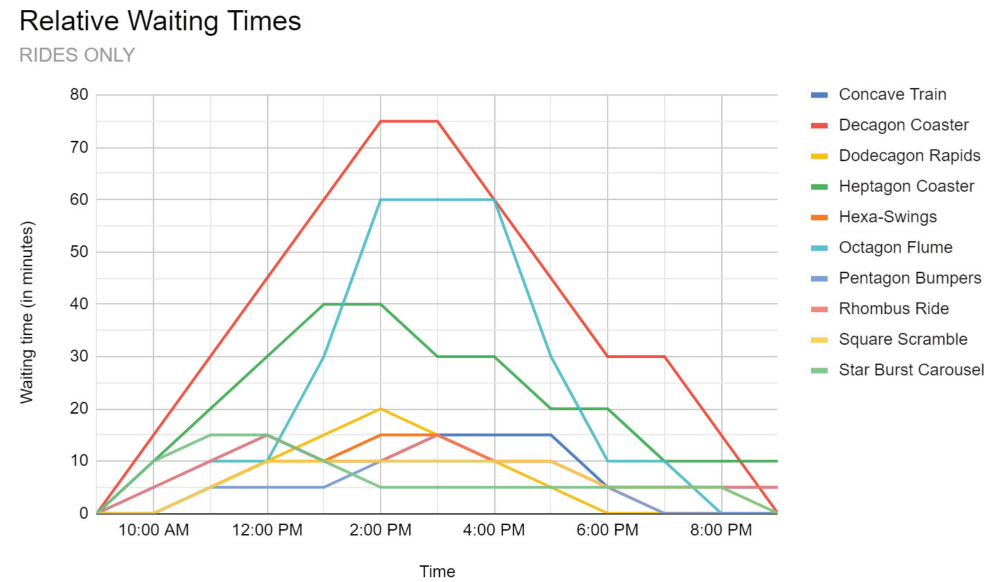
The variables we considered as shown in table 1:

Table 1 Mathematical model analysis variables definition

Variable	Description
H_i	Happiness score, i —-- team member
C_j	Coefficient of happiness, $C_j \in [-2,2]$
$t_{activity}$	Time spent on $activity$ (min)
t_{wait}	Waiting time (minutes)
t_{walk}	Walking time (minutes)
t_{buffer}	Buffer time (minutes)
α_k	$\alpha_k \in [0,0.6,1]$ $\alpha_k = 1$ if requirement is met $\alpha_k = 0$ if requirement is not met $\alpha_k = 0.6$ is when the requirement stated by the question is not met. For example, separating the friends for more than 4 hrs a day.
D	Walking distance (cm)

Example 4: Variables Problem C,
Team 13338, NUS High School of Math and Science, Singapore

4.3 The Schedule



[1]

First, one has to consider what the first event is. Of the rides mentioned, the ride to prioritize is the Decagon Coaster because it is the ride with the longest wait times, which is shown in the graph above. To avoid longer wait times later, it is optimal to ride it first. While we know Ming gets motion sick on rides, it is not optimal for the group to split up for only one activity. So the group will go together, and Ming could sit this ride out.

If they rush to the Decagon Coaster, they can be among the first in line and thus avoid the long wait time. This will take the group approximately three minutes. We calculated this by using the formula for speed.

$$s=d/t$$

Or

[1] This chart was made in Google Sheets with the information given in the problem.

$$t=d/s$$

Since we know the distance and speed all we can plug it into the equation, and convert our answers into minutes.

$$t = 222.25\text{m}/1.3\text{s}$$

$$t \approx 171s \approx 3min$$

Then, we have to find the wait time at 9:03. From assumption one, we are assuming the wait time increases and decreases linearly. The initial wait time is 0 minutes, and the final time at the end of the hour is 15 minutes. Then will find the equation of this segment that represents the wait time of the hour. If we use the slope intercept form :

$$y=mx+b$$

x represents the time in minutes, for example, if the time is 10:05, x would be 5 since the hour can be omitted from the calculation of the slope. We can omit it because we calculate the slope of the hour, so the hour would stay the same. y represents the wait time.

To find the slope, we can use :

$$m=\frac{y_2-y_1}{x2-x1}$$

We can plug the numbers in, m = (0-15)/(0-60) =-15/-60 = ¼, which is the slope. The y-intercept is 0, so the equation will be: $y=(¼)x+0$. If we plug the time in, 3 minutes, so y=¾ which is .75 minutes, and we will round to 1 minute. Thus they will start at 9:04, and with a ride time of 3 min they will be finished at approximately 9:07. This method is how we calculated the wait times of the other rides at a specific time.

Example 5: Model Development Problem C, Continued
Team 13296, Princeton Charter School, USA

1. A letter to my friends

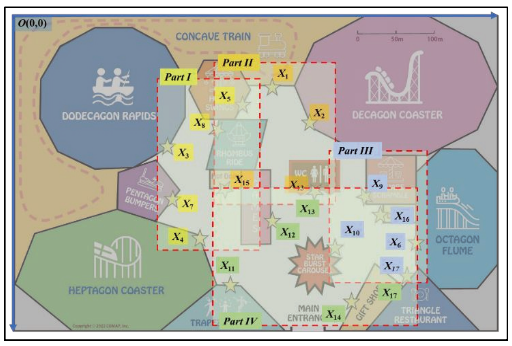
Dear Ming、Ishmael、Karine、Freya，

Thanks for hiring our team at the MidMCM Travel Agency to develop a schedule for your one-day trip to the Polygon Paradise Park. We will try our best to provide you an ideal day guide at the park. Let's Familiarize with Polygon Paradise Park at first.

Introduction to the Park

Polygon Paradise Park is a small amusement park open from 9am to 9pm. The park has ten rides of different types and thrill levels. The Trapezoid Show, a location with several different performances throughout the day, offers a circus acrobatics show, a magic show, and evening fireworks. The Games building has both arcade and carnival games. Food options include the Triangle Restaurant, the hot dog stand, and the ice cream cart. The hot dog stand and the ice cream cart are "take away" options, while the Triangle Restaurant requires 30-60 minutes to sit at a table and enjoy a meal. The restrooms (WC) are centrally located in the park for easy access. Visitors can purchase souvenirs in the gift shop located near the park entrance.

Ideal day guide at Polygon Paradise Park



Map of the Park

Part	Start Time	Location	Wait Time	Time taken	Next Location	Walking Time	End Time	Time	Tips
Part I 09:00 -10:00	9:00	X4	0:10	0:02	X7	0:02	9:14	09:00-10:00	Heptagon Coaster
	9:14	X7	0:00	0:15	X3	0:02	9:31	09:00-10:00	Pentagon Bumpers
	9:31	X3	0:00	0:04	X8	0:02	9:37	09:00-10:00	Dodecagon Rapids
	9:37	X8	0:05	0:03	X5	0:01	9:46	09:00-10:00	Rhombus Ride
	9:46	X5	0:00	0:05	X1	0:01	9:52	09:00-10:00	Hexa-Swings
Part II 10:00 -11:00	9:52	X1	0:05	0:03	X13	0:03	10:03	09:00-10:00	Concave Train
	10:03	X2	0:30	0:03	X15	0:03	10:39	10:00-11:00	Decagon Coaster
	10:39	X15	0:05	0:07	X13	0:03	10:54	10:00-11:00	Hot Dog
	10:54	X13	0:00	0:06	X9	0:02	11:02	10:00-11:00	WC
Part III 11:00 -14:00	11:02	X9	0:10	0:10	X16	0:01	11:23	11:00-12:00	Square Scramble
	11:23	X16	0:05	0:25	X6	0:02	11:55	11:00-12:00	Icecream
	11:55	X6	0:10	0:03	X17	0:01	12:09	11:00-12:00	Octagon Flume
	12:09	X17	0:30	1:00	X10	0:02	13:41	12:00-13:00	Lunch
	13:41	X10	0:05	0:05	X11	0:03	13:54	13:00-14:00	Star Burst Carousel
Part IV 14:00 -21:00	13:54	X11	0:06	1:00	X12	0:03	15:03	14:00-15:00	Magic Show
	15:03	X12	0:00	0:54	X11	0:03	16:00	15:00-16:00	Games
	16:00	X11	0:00	1:00	X14	0:05	17:05	16:00-17:00	Circus Acrobatics
	17:05	X14	0:00	0:53	X17	0:02	18:00	17:00-18:00	Gift Shop
	18:00	X17	0:10	1:00	X13	0:04	19:14	18:00-19:00	Dinner
	19:14	X13	0:00	0:42	X11	0:04	20:00	19:00-20:00	WC
	20:00	X11	0:00	1:00	/	0:00	21:00	20:00-21:00	Firework Show

Schedule of the one-day trip

Now let's take a look at the schedule of this trip.

- First, this park will be divided into 4 parts and including 17 locations in our schedule. You will start the one-day trip at the location of Heptagon Coaster. Please arrive at this location at 9am on time. You will play in the park clockwise to minimize the waiting time and walking time.
- You will have lunch about 12:15, and you can buy hot dogs as snacks. Considering Karine doesn't like hot dogs very much, we suggest that she can bring her own snacks.
- After lunch, most of the queues in all attractions are already more than 30minutes. The afternoon and the late afternoon are the time for shows, games, mini-attraction, or some photo opportunities in the garden.

Example 6: Letter and Plan Problem C,
Team 12704, WenZhou Nanpu Experimental Middle School, China

- You will have dinner about 18:00, After your stomach is feeling good and you are feeling good, you can stick around and watch the fireworks show.
- After the fireworks show, it's time to say goodbye. We have completed the exploration of all projects. I hope you are satisfied with this schedule.

Highlights of the Schedule

There are five highlights in our schedule.

- Arrange the right project at the right time to reduce the waiting time. According to our schedule, the average waiting time for completing all 10 rides only 10 minutes.
- During this day's trip, time will not waste on walking. The total walking time will be less than 1 hour. The longest walking time shall not exceed 5 minutes.
- The schedule includes enough leisure time, which can be used to have a meal, take a rest, play games or buy souvenirs.
- At the same time, we will also make targeted arrangements according to the different needs of everyone to make the collective satisfaction as high as possible.
- All rides and shows will be completed, and individual rides can be tried more than once.

Four tips

It is very important to know the following tips.

- Pack light and wear sneakers. Before you go, think about what you absolutely need to bring with you. There are lockers at every roller coaster to drop your bag, but it might slow you down. And don't wear flip flops. On rides like Decagon Coaster, you will have to take them off. Plus, sneakers will help you get from one location to the other as fast as possible.
- Follow the rules, not the other kids. The rules are to help you have fun, not spoil it.
- Pay attention to specific safety instructions. Watch out when the rules say things like "hang on to the handles," "slide only feet first," "stay seated," "don't rock the seat," "get rid of gum before you ride" or "no flipping." When they post those rules, it is because they know those actions can cause kids to get hurt.
- Read the signs and teach your parents some safety rules. Even adults get hurt when they put their hands in the air or don't ride properly.

The final last, buy yourself a t-shirt to celebrate and a souvenir of your accomplishment! I hope you'll have fun and make the best out of your time.

Example 6: Letter and Plan Problem C, Continued
Team 12704, WenZhou Nanpu Experimental Middle School, China

6.4 Notes for each one

1.Notes to Ming

- The game time is scheduled at 15:00-16:00, about 1 hour of free time.
- All the extremely Thrill rides (X_4, X_8, X_2) are arranged before lunch, you can decide whether to participate in these rides according to your own situation.
- The waiting time of X_2 is relatively long, and the project is more exciting, you can go to Games instead. But be sure to wait by 10:39 at the hot dog stand, which is next door to the Games.

2. Notes to Ishmael

- We had nearly the entire afternoon to watch the show and could see three different shows. The time of the magic show is scheduled for 14:00-15:00 in the afternoon.
- There are three extremely thrill roller coaster rides in the park, we've all got them on the schedule.
- If you want to participate in roller coasters repeatedly, you can invite the team to play them together during the break. Available time is 17:00-18:00 or 19:00-20:00, the wait times for those times are very short. But you must not miss the time to go to the next location with the team.

3.Notes to Karine

- The good news for you is that most locations have relatively short wait times. All items except X_2 and X_{17} will have a wait time of less than 10 minutes.
- The time of the circus acrobatics show is scheduled for 16:00-17:00 in the afternoon.
- The lunch time is 12:00-13:00, and the dinner lunch is 18:00-19:00. We suggest that you can bring yourself some snacks. Except for hot dogs, there are no other snacks that can be eaten in this park.

4.Notes to Freya

- There are two water rides in the park, namely Dodecagon Rapids and Octagon Flume. Compared with other rides, their play time is relatively long, I hope you enjoy yourself.
- Our schedule allows the group to be together for most of the day. Even during free time, best friends can play some fun games into groups of two.
- The shopping time is scheduled at 17:00-18:00 at gift shop, about 1 hour of free time. Hope you find meaningful souvenirs then. In addition, bringing a camera and capturing some beautiful moments is also a good way to add meaning to the trip.

Example 7: Additional Detail Problem C,
Team 12704, WenZhou Nanpu Experimental Middle School, China

5 Analysis

5.1 Strengths

Like any model, our method has strengths and weaknesses. One of our model's strengths is that it is extremely easy to implement and has a simple yet logical structure. Similar to many powerful chess engines, our model does not consider all possible "moves," but rather only the ones deemed the most likely to be optimal. We do this by only considering options that yield an event in which our variables are close to optimal (i.e facilitating short distances and wait times and conforming to the groups preferences).

Another strength of our solution is that it considers everyone's preferences equally in a ranked and organized manner that provides for an equally enjoyable experience for everyone.

Among the specific strengths of our solution are:

- We organized the waiting time for the rides relatively well. This allows the group to conveniently visit all the rides in time.
- We arranged for the group to watch their preferred shows at times when the general waiting times were the highest.
- The group is split up effectively and makes good use of the allotted time
- The group only split up for around 40 minutes, and they spent most of the time together, which was a preference of Freya.
- The group did all the rides twice, except the heptagon and octagon coaster, which proves they made good use of their time and enjoyed all the park had to offer.
- The group was able to go to all the activities they wanted to do.

Example 8: Strengths and Limitations Problem C, Team 13296, Princeton Charter School, USA

5.2 Weaknesses

Our model's weakness is that while it produces a fairly good schedule it obviously does not produce the ideal schedule, and one does not know how close it is to the ideal schedule. Also, while as previously mentioned it is very simple to implement, it is slow to apply without the aid of a coded version of the algorithm. Another weakness is that given the limitations of the given data our solution makes extensive use of various assumptions and does not assign numerical values to what is given and what can be deduced.

Among the specific weaknesses of our solution are:

- Ming doesn't split off from the group in the first ride, which is a roller coaster. Ming would get motion sickness, and likely wouldn't enjoy this ride. However, it is not efficient for them to split up at this time, so we have to prioritize the rest of the group over Ming, and she could even choose to sit the ride out.
- Ishmael would probably like to go on the roller coasters several more times.
- The group did not visit the Hot Dog stand, as Karine does not like hotdogs.
- The group

Example 8: Strengths and Limitations Problem C, Continued
Team 13296, Princeton Charter School, USA

7 Our Schedule

The above schedule is great for Ming, Ishmael, Karine, and Freya. However, if our group went to Polygon Paradise, our schedule would look different since we have different preferences. Two of our group members don't enjoy water rides, and one can't eat ice-cream. One of our members loves roller coasters, one loves fireworks, and two would love to go to the carousel. All members don't want to go to the arcade, so unlike Ming and the previous group, we will not make a stop at the arcade during the day. On top of that, our group wouldn't want to split up since we would all find it more enjoyable with friends, so we would have to make compromises and might have to sit out on a couple rides. We are assuming all other variables (walking speed, distance, and wait times) would stay constant. Our schedule would consist of starting at Heptagon Coaster and going around the park, skipping the arcade and water rides, to minimize walk time. During lunch time, we would spend an hour at the triangle restaurant, since that is the minimum amount of time we would like to spend in a restaurant to have full enjoyment. We would then have a bathroom break and try the other rides, riding on the carousel a couple of times when the wait time is low. During our trip, Our group would end the day looking at the beautiful fireworks show.

Just like we applied this model to create a schedule for our group, this model can be applied to create a schedule with different parameters. Using the same method of considering the next optimal event based on preferences, distance, and wait times, we can develop a schedule regardless of the specific parameters.

Example 9: Reflection Problem C,
Team 13296, Princeton Charter School, USA

Chapter 4 Conclusion and future development

Conclusion:

Overall, we have accommodated the needs and constraints of the four friends and have made an optimal schedule for each of them. We have accommodated their needs and made a Gantt Chart for each of them. According to our analysis, we can conclude:

- Gantt Chart is an efficient method for scheduling.
- Happiness mathematical model can be applied for the assessment of scheduling.
- We have also compared the schedule we designed for the four friends with the schedule we would have designed for ourselves.
- Even though we have done the job required, there is always space for future development. Such as making the Gantt chart in 3d.

Future Development:

- Apply 3D Gantt Charts for a more complex scheduling [3]
- Apply CPLEX tool for this scheduling problem and find the optimum solution
- Plan multiple cases with different objectives such as walking time, activity time, waiting time while fulfilling all the requirements stated by the question and the group of friends.

Example 10: Conclusion Problem C,
Team 13338, NUS High School of Math and Science, Singapore